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Genetic Algorithm Based Goal Programming Procedure for Solving Interval-Valued Multilevel Programming Problems

Papun Biswas[1], Bijay Baran Pal[2], Anirban Mukhopadhyay[3], Debjani Chakraborti[4]
Department of Electrical Engineering, JIS College of Engineering, Kalyani-741235, West Bengal, India[1]
Department of Mathematics, University of Kalyani, Kalyani-741235, West Bengal, India[2]
Department of Computer Science & Engineering, University of Kalyani, Kalyani-741235, West Bengal, India[3]
Department of Mathematics, Narula Institute of Technology, Kolkata-700109, West Bengal, India[4]

Abstract

This article presents goal programming (GP) procedure for solving Interval-valued multilevel programming (MLP) problems by using genetic algorithm (GA) in a hierarchical decision making and planning situation of an organization. In the proposed approach, first the individual best and least solutions of the objectives of the decision makers (DMs) located at different hierarchical levels are determined by using the GA method. Then, the target intervals of each of the objectives and decision vectors controlled by the upper-level DMs are defined in the inexact decision environment. Then, in the model formulation, the interval valued objectives and control vectors are transformed into the conventional form of goal by using interval arithmetic technique. In the goal achievement function, both the aspects of minsum and minmax GP formulations are adopted to minimize the lower bounds of the defined regret intervals for goal achievement within the specified interval from the optimistic point of view of the DMs.The potential use of the approach is illustrated by a numerical example.

Keywords

Multiobjective decision making, Multilevel programming, Goal programming, Interval programming, Genetic algorithm

1. Introduction and Literature Review

MLP is a special field of study in the area of mathematical programming (MP) for solving decentralized planning problems having multiple DMs with different objectives in a large hierarchical decision organization. In multilevel programming problems (MLPPs) [1], one DM is located at each of the different hierarchical levels and each control a decision vector and an objective function separately in the decision making context. The execution of the decision power is sequential from an upper-level to a lower-level and each DM tries to optimize his/her own benefit paying serious attention to the benefits of the others in the decision environment.

It may be mentioned here that most organizations of today have multilevel hierarchical decision structures. In a practical decision situation, although the execution of the decision powers is sequential, the decision of an upper-level DM is often affected by the reactions of lower-level DMs due to their dissatisfaction with the decision of the upper-level DM. As a consequence, most of the hierarchical decision organizations frequently face the problem of proper distribution of decision powers to DMs for overall benefits of the organizations.

Now, in the field of multiobjective decision making (MODM), the concept of MLPP for solving decentralized planning problems of a large decision system was suggested in [2]. Thereafter, during the past three decades, various versions of MLPPs as well as bilevel programming (BLP) problems, as a special case of MLPPs, have widely been studied extensively in [3,4,5,6,7] by the pioneer researchers in the field from the view point of their potential applications of such approaches to different real-life hierarchical decentralized problems, viz., firm management [8] , economic policy system [9], manufacturing [10], and especially for conflict resolutions [11].

Most of the classical approaches for solving hierarchical decision problems developed so far in the past often lead to the paradox that the decision power of a higher level DM is dominated by a lower level DM.

However, concerning the hierarchical decision structure of a decentralized system, it is generally assumed that the DMs cooperative each other to reach a minimum level of satisfaction of each of them for smoothing the activities of the organization. In such a situation, in order to overcome the shortcomings of the classical approaches, the idea of fuzzy programming (FP) [12] based on the concept of fuzzy set theory [13] has been introduced to solve hierarchical decision problems.

But, the main drawback of such approaches is that there is a possibility of rejecting the solution again and again by the leader and re-evaluation of the problem with the elicited membership values of the membership functions is repeatedly introduced in the solution search process due to conflict in nature of the objectives. As a result decision deadlock often arises and the problem of proper distribution of decision powers is encountered in a decision making situation.

To avoid such a computational difficulty, fuzzy goal programming (FGP) approach [14] as an extension of conventional GP [15] based on the goal satisficing philosophy [16], has been studied in [17] for making decision with regard to achievement of multiple fuzzy goals in uncertain environment.

Now, it may be noted that although FP as well as FGP have been successfully implemented to the MODM problems, the main difficulty of using such approaches is that it may not always be possible for the DM to assign the fuzzy aspiration levels to the objectives and thereby defining the tolerance ranges of goal achievement in highly sensitive decision situations. Sometimes, tolerance intervals defined for achievement of the fuzzy goals are found not proper to arrive at a satisfactory decision.

To overcome the above difficulty, interval programming (IvP) approaches [18] have appeared as the prominent tool for solving MODM problems with interval-valued parameter sets in an inexact decision environment. The IvP approaches are actually based on interval arithmetic [19] technique for achieving the solution of decision problems with interval-valued parameter sets. IvP approaches to decision problems in inexact environment have been deeply studied in [20] in the past. The basic difference between the IvP and FP approaches in the imprecise decision premises is that a satisfactory decision can always be made by specifying the intervals on the basis of needs and desires of the DMs in the decision situation in the first case, whereas a decision trouble may occur in the latter case if there is a lack of proper setting of imprecise parameter values in the decision-making environment. However, the judgement of superiority of one over the other can be made depending on the decision-making context.

Now, two types of methodological aspects are used to solve the IvP problems. The first one is based on the satisfying philosophy of GP and second one is on the traditional method of optimization. The GP formulation in MODM problems with interval parameter sets has been introduced in [21]. The solution approach with *minimax* regret criteria for obtaining the two types of solutions, necessary and possibly optimal solutions, has been investigated in [22] in the past. The methodological development of IvP has been surveyed in [23]. The IvP methodology for multiobjective fractional programming problems has been studied in [24]. However, methodological extension of IvP is still at an early stage from the viewpoint of its use to different real-life problems. Again, the IvP approach to MLPPs in the field of hierarchical optimization is yet to be circulated in the literature.

Now, in order to solve most of the real-world hierarchical decision problems in the past, the conventional linear approximation approaches [25] were used, which involve huge computational load and inherent approximation errors in the decision search process.

To overcome the computational difficulties arising due to the use of these traditional (single-point based) solution search approaches, GAs based on the natural selection and population genetics, initially introduced in [26] have appeared as volume-oriented global solution search tools to solve complex real-world MODM problems. The deep study on GA based solution methods to different problems has been well documented in [27, 28] in the past.

The extensive study on the use of GAs as goal satisficers rather than objective optimizers to multiobjective decision problems in crisp decision environment has been discussed in [29]. The efficient use of GA solution search methods for solving conventional GP models of nonlinear MODM problems have also been studied in [30] with a view to avoid computational load for linearizing objectives in the process of solving the problems. The potential use of GAs to FGP models of real-life problems have been studied [31] in the recent past.

The GA based solution approach to bilevel programming problems (BLPPs) in crisp decision environment was first studied in [32]. Thereafter, the computational aspects of GAs to fuzzily described hierarchal decision problems have been investigated in [33] in the past. The GA based solution concept to FGP approach has been further extended for MLPPs [34] in the past.

The robustness of GA based GP approach to interval-valued BLPPs has been investigated in [35]. However, methodological extension of using GAs to IvP problems is still at an early stage and their application to real-world problems is rare in literature. Further, deep study of the use of IvP approach to MLPPs in the field of hierarchical decision problems is yet to appear in the literature.
In the proposed approach, first the objectives at the different hierarchical levels with interval coefficients are converted into interval-valued goals by introducing target intervals for achievement of the objectives to certain levels of their satisfactions.

Then the interval-valued objectives are transformed into objective goals in the standard GP formulation by using interval arithmetic technique and introducing under- and over- deviational variables to each of them. In the model formulation of the problem, both the aspects of GP, *minsum* GP [15] and *minmax* GP [36] are taken into consideration to construct the achievement function for achievement of the objective values within the target specified in the decision situation.

In the solution process, the GA scheme is introduced first to define first the target intervals of the objectives. Then, goal achievement of the model goals is constructed, which is considered as the evaluation function of the GA scheme.
A numerical illustration is provided to expound the potential use of the approach.

2. Interval-Valued MLPP Problem Formulation

Let $X = (x_1, x_2, \ldots, x_n)$ be the vector of decision variables involved with different levels of the decision system. Then, let Z_k, k = 1,2,…,K be the objective function and X_k be the control vector of the decision variables of the k-th level DM, k=1,2,…,K, $K \le n$, $X \in R^n, X_k = (x_{k1}, x_{k2}, \ldots, x_{kn_k}) \in R^{n_k}$ where R^n $(n = n_1 + n_2 + \ldots + n_K)$ is the n-dimensional Euclidean space. Then, the MLPP with interval coefficients can be stated as:

Find $X = (X_1, X_2, \ldots, X_K)$ so as to

$$\underset{X_1}{\text{maximize}}\, Z_1(X) = \sum_{k=1}^{K} [C_{1k}^L, C_{1k}^U] X_k$$

(top-level problem)
where, for given $X_1; X_2, X_3, \ldots, X_K$ solve

$$\underset{X_2}{\text{maximize}}\, Z_2(X) = \sum_{k=1}^{K} [C_{2k}^L, C_{2k}^U] X_k$$

(second-level problem)
where, for given X_1 and $X_2; X_3, \ldots, X_K$ solve

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where, for the given $X_1, X_2, \ldots X_{K-1}; X_K$ solves

$$\underset{X_K}{\text{maximize}}\, Z_K(X) = \sum_{k=1}^{K} [C_{Kk}^L, C_{Kk}^U] X_k$$

(the K-th level problem)

(1)

subject to

$$X \in S = \left\{ X \in R^n \middle| AX \begin{pmatrix} \ge \\ \le \end{pmatrix} b, X \ge 0, b \in R^m \right\}, \quad (2)$$

where X is a vector of decision variables, $[C_{jk}^L, C_{jk}^U]$, (j, k=1,2,…,K) are the vector of interval coefficients, L and U denote the lower and upper bounds, respectively, of the defined interval, A is a constant matrix and b is a constant vector. It is assumed that the feasible region $S(\ne \varphi)$ is bounded.

Now, using the rules of interval arithmetic operations [19], the interval-valued objectives in (1) can be successively expressed as:

$$\begin{aligned}\underset{X_1}{\text{maximize}}\, Z_1 &= \sum_{i=1}^{n_1} [c_{11}^{L_i}, c_{11}^{U_i}] x_{1i} + \sum_{i=1}^{n_2} [c_{12}^{L_i}, c_{12}^{U_i}] x_{2i} + \ldots + \sum_{i=1}^{n_K} [c_{1K}^{L_i}, c_{1K}^{U_i}] x_{Ki} \\ &= [\sum_{i=1}^{n_1} c_{11}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{L_i} x_{Ki}, \\ &\quad \sum_{i=1}^{n_1} c_{11}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{U_i} x_{Ki}]\end{aligned}$$

(top-level problem)

$$\underset{X_2}{\text{maximize}}\, Z_2 = \sum_{i=1}^{n_1}[c_{21}^{L_i}, c_{21}^{U_i}]x_{1i} + \sum_{i=1}^{n_2}[c_{22}^{L_i}, c_{22}^{U_i}]x_{2i} + \ldots + \sum_{i=1}^{n_K}[c_{2K}^{L_i}, c_{2K}^{U_i}]x_{Ki}$$
$$= [\sum_{i=1}^{n_1} c_{21}^{L_i}x_{1i} + \sum_{i=1}^{n_2} c_{22}^{L_i}x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{L_i}x_{Ki}, \sum_{i=1}^{n_1} c_{21}^{U_i}x_{1i} + \sum_{i=1}^{n_2} c_{22}^{U_i}x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{U_i}x_{Ki}]$$

(second-level problem)

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$$\underset{X_K}{\text{maximize}}\, Z_K = \sum_{i=1}^{n_1}[c_{K1}^{L_i}, c_{K1}^{U_i}]x_{1i} + \sum_{i=1}^{n_2}[c_{K2}^{L_i}, c_{K2}^{U_i}]x_{2i} + \ldots + \sum_{i=1}^{n_K}[c_{KK}^{L_i}, c_{KK}^{U_i}]x_{Ki}$$
$$= [\sum_{i=1}^{n_1} c_{K1}^{L_i}x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{L_i}x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{L_i}x_{Ki}, \sum_{i=1}^{n_1} c_{K1}^{U_i}x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{U_i}x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{U_i}x_{Ki}]$$

(the K-th level problem)

(3)

where, $[c_{jk}^{L_i}, c_{jk}^{U_i}]$ are the components of the vector of interval coefficients $[C_{jk}^{L}, C_{jk}^{U}]$, (j, k=1,2,…,K).

Now, in the field of IvP, instead of introducing a single target level as the aspiration level (crisp or fuzzy) for goal achievement, an interval-valued target called the target interval [19,21] for the possible goal achievement is considered in the inexact decision environment. In the present decision situation, since the objectives at different hierarchical levels are conflicting in nature due to the hierarchy of execution of decision powers, and each of the DMs desire to obtain a solution to a certain satisfactory level, an GA method as the goal satisficer [29] rather than objective optimizer can be considered as an efficient multiobjective tool for solving hierarchical decision problems.

The GA scheme adopted in the solution search process is presented in the following Section 3.

3. Design of the GA Scheme

In the literature of GAs, there is a number of schemes [26, 27] for generation of new populations with the use of the different operators: selection, crossover and mutation. Here, the binary coded representation of a candidate solution called chromosome is considered to perform genetic operations in the solution search Process. The conventional Roulette wheel selection scheme in [27], single-point crossover [28] and bit-by-bit mutation operations are adopted to generate offspring in new population in search domain defined in the decision making environment.

The fitness score of a chromosome *v* (say) in evaluating a function, say, eval (E_v), based on maximization or minimization of an objective function defined on the basis of DMs' needs and desires in the decision making context.

The fitness value of each chromosome is determined by evaluating an objective function.

The fitness function is defined as:

$$\text{eval } (E_v) = (Z_k)_v, \; k=1,2,\ldots,K; \; v=1,2,...,\text{pop\_size} \quad (4)$$

where Z_k represents the objective function of the k-th level DM given by (1) and where the subscript 'v' in (4) refers to the fitness value of the selected v-th chromosome, v = 1, 2, ..., pop_size.
The best chromosome with largest fitness value at each generation is determined as:

$$E^* = \max\{\text{eval } (E_v) \mid v = 1, 2, ..., \text{pop\_size}\}$$
or
$$E^* = \min\{\text{eval } (E_v) \mid v = 1, 2, ..., \text{pop\_size}\},$$

which depends on searching the maximum or minimum value of an objective function.

Now, the model formulation of interval-valued MLPP is presented in the next Section 4.

4. Interval-valued MLPP model formulation

To represent the objectives in the form of interval-valued goals, the target intervals for each of them is to be introduced by defining the lower- and upper-bounds of the intervals for achieving the objectives within a permissible range in the decision making context.

In the above situation, the best and least solutions of the individual objectives can be reasonably considered as the upper- and lower-bounds of the target intervals for possible goal achievement in the MODM situation.

Here, the proposed GA method can be used to determine the solutions by defining the GA parameter values.

Let the individual best and least solutions of the j-th objective be $(X_1^{jb}, X_2^{jb}, \ldots, X_K^{jb}; T_j^{*b})$ and $(X_1^{jl}, X_2^{jl}, \ldots, X_K^{jl}; T_j^{*l}),\ j = 1,2,\ldots,K$, respectively, where $T_j^{*b} = \max_{X \in S} Z_j^U,\ j = 1,2,\ldots,K$ and $T_j^{*l} = \min_{X \in S} Z_j^L,\ j = 1,2,\ldots,K$ and where

$$Z_j^U = \sum_{i=1}^{n_1} c_{j1}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{j2}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{jK}^{U_i} x_{Ki},\ j = 1,2,\ldots,K$$

and

$$Z_j^L = \sum_{i=1}^{n_1} c_{j1}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{j2}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{jK}^{L_i} x_{Ki},\ j = 1,2,\ldots,K$$

Now, in the decision making context, it is reasonably assumed that the DMs are motivated to cooperate each other and each is willing to sacrifice his/her own benefit up to a certain level for a gain of the other from the view point of survival as well as sustainable growth of the organization.

In the above context, the target intervals of the objectives $Z_k, k = 1,2,\ldots,K$ can be determined as

$[t_j^{*l}, t_j^{*b}],\ j = 1,2,\ldots,K$

where, $T_j^{*l} \le t_j^{*l} \le t_j^{*b} \le T_j^{*b},\ j = 1,2,\ldots,K$

(5)

and the consideration of which depends on the decision making situation.

The objectives in (3) with interval coefficients and target intervals successively appear as:

$$[\sum_{i=1}^{n_1} c_{11}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{L_i} x_{Ki},$$
$$\sum_{i=1}^{n_1} c_{11}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{U_i} x_{Ki}] = [t_1^{*l}, t_1^{*b}]$$

(top-level problem)

$$[\sum_{i=1}^{n_1} c_{21}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{22}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{L_i} x_{Ki},$$
$$\sum_{i=1}^{n_1} c_{21}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{22}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{U_i} x_{Ki}] = [t_2^{*l}, t_2^{*b}]$$

(second-level problem)

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$$[\sum_{i=1}^{n_1} c_{K1}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{L_i} x_{Ki},$$
$$\sum_{i=1}^{n_1} c_{K1}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{U_i} x_{Ki}] = [t_K^{*l}, t_K^{*b}]$$

(K-th level problem)

(6)

Now, since execution of decision powers of the DMs is sequential, and the objectives generally conflict each other due to the interest of each of the DMs regarding achievement of the objective values to the highest possible one, certain relaxations on the decisions $X_k^{jb}, k = 1,2,\ldots,K-1$ of the upper-level DMs need to be given for the benefit of the lower-level DM.

Let $X_k^p (X_k^{jl} < X_k^p < X_k^{jb}), k = 1,2,\ldots,K-1,$ be the lower tolerance limit of the decisions X_k, k= 1, 2, … , K-1) of the upper-level DMs.

Using the notion of mid-point arithmetic in IvP [19], the decision X_k with target interval appears as:

$$X_k = [X_k^p, X_k^{jb}] \quad (7)$$

Now, the GP formulation of the problem is discussed in the following Section 5.

5. GP Model formulation of interval-valued MLPP

To formulate the GP model of the problem, the objectives in (5) are transformed into the standard goals by using the interval arithmetic operation rule [21] and introducing the under- and over- deviational variables to each of them. The conversion of interval-valued goals to the conventional form of the goals for the successive decision levels are obtained as:

$$\sum_{i=1}^{n_1} c_{11}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{U_i} x_{Ki} + d_{1L}^- - d_{1L}^+ = t_1^{*l}$$

$$\sum_{i=1}^{n_1} c_{11}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{12}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{1K}^{L_i} x_{Ki} + d_{1U}^- - d_{1U}^+ = t_1^{*b}$$

(top-level problem)

$$\sum_{i=1}^{n_1} c_{21}^{U_i} x_{1i} + \sum_{i=}^{n_2} c_{22}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{U_i} x_{Ki} + d_{2L}^- - d_{2L}^+ = t_2^{*l}$$
$$\sum_{i=1}^{n_1} c_{21}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{22}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{2K}^{L_i} x_{Ki} + d_{2U}^- - d_{2U}^+ = t_2^{*b}$$

(second-level problem)

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$$\sum_{i=1}^{n_1} c_{K1}^{U_i} x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{U_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{U_i} x_{Ki} + d_{KL}^- - d_{KL}^+ = t_K^{*l}$$
$$\sum_{i=1}^{n_1} c_{K1}^{L_i} x_{1i} + \sum_{i=1}^{n_2} c_{K2}^{L_i} x_{2i} + \ldots + \sum_{i=1}^{n_K} c_{KK}^{L_i} x_{Ki} + d_{KU}^- - d_{KU}^+ = t_K^{*b}$$

(the K-th level problem) (8)

where, $d_{kU}^-, d_{kL}^- \geq 0, k = 1,2,\ldots,K$, represent under- deviational variables and $d_{kU}^+, d_{kL}^+ \geq 0, k = 1,2,\ldots,K$ represent over-deviational variables of the respective goal expressions.

Similarly, the standard goal expressions for the decision vectors $X_k, k = 1,2,\ldots,K-1$ are obtained as:

$$X_k + d_{rL}^- - d_{rL}^+ = X_k^p \text{ and}$$
$$X_k + d_{rU}^- - d_{rU}^+ = X_k^{jb},\ r = K + k;\ k = 1,2,\ldots,K-1 \quad (9)$$

where, $d_{rL}^-, d_{rL}^+ (\geq 0)$ and $d_{rU}^-, d_{rU}^+ (\geq 0)$ represents the vectors of under- and over- deviational variables, and dimension of them depends on the dimension of X_k.

Now, in the decision making environment, the aim of the DMs is to minimize the possible regrets for under-deviations from t_j^{*l} and over-deviations from $t_j^{*b}, j = 1,2,\ldots,K$ and also the regrets for vectors of under- and over-deviations from X_k^p and X_k^{jb}, respectively, defined for the upper level DMs' control vectors $X_k, k = 1,2,..,K-1$ to reach a satisfactory solution for achieving the aspired goal levels within their respective target intervals specified in the decision situation. As a matter of fact, minimization of the deviational variables d_{kL}^- and d_{kU}^+, (k =1, 2, …, K) and the vectors of deviational variables d_{rL}^- and d_{rU}^+ (r = K+1, K+2,…, 2K-1) in the context of minimizing the total possible regrets for goal achievement is taken into account, and then the designed objective function of the proposed GP model is termed as the 'regret function' in the sense of minimizing the regrets.

The development of the executable GP model by constructing the regret function is presented as follows:

5.1. Construction of the executable GP model

There are different versions of GP [37,38] for solving real-life MODM problems. The most widely used approaches are weighted GP (WGP) [36], *minmax* GP (MGP) [39], and non-dominance GP [15].

In WGP, also called *minsum* GP, minimization of the (unwanted) deviational variables in the achievement function (regret function) is considered on the basis of weights of importance of achieving the target levels of goals in the decision environment.

On the other hand, in case of MGP, minimization of maximum deviation of a goal from the target level is considered. This approach provides a solution that gives highest importance to the goal most displaced with respect to its target. Here, the most balanced solution among the achievement of different goals is obtained.

An intuitive idea in using GP is to take the convex combination of WGP and MGP models, called extended GP (EGP) [40], to make a reasonable balance of the solution for aggregate achievement of goals provided by the former model and a balanced solution provided by the latter one.

In a GP model, the objective function is termed as 'achievement function' where minimization of the (unwanted) deviational variables on the basis of needs and desires of the DMs is considered in the decision making context.

In the present context of GP formulation of the problem, both the stated aspects of GP, WGP and MGP, are taken together as their convex combination to reach a satisfactory decision within the target intervals defined for the problem in the decision situation.

Now, from the optimistic point of view of all the DMs, minimization of the possible regrets involved with both the under- and over- deviational variables d_{kL}^- and $d_{kU}^+, (k = 1,2,\ldots,K)$ as well as the vector of deviational variables d^-_{rL} and d^+_{rU} (r = K+1, K+2, …, 2K-1) are taken into consideration to reach a reasonable balance of decision powers of the DMs in the decision making environment.

The regret function appears as:

$$\text{minimize}\, Z = \lambda\{\sum_{k=1}^{K}(w^-_{kL}d^-_{kL} + w^+_{kU}d^+_{kU}) + \sum_{r=K+1}^{2K-1}(w^-_{rL}d^-_{rL} + w^+_{rU}d^+_{rU})\} + (1-\lambda)D, 0 < \lambda < 1 \quad (10)$$

where $D = \max_{m=1}^{2K-1}\{(d^-_{mL} + d^+_{mU})$.

$w^-_{kL}, w^+_{kU} \geq 0$ (k = 1,2,…,K) and $w^-_{rL}, w^+_{rU} \geq 0$ (r = K+1, K+2,…, 2K-1) represent the relative numerical weights of importance of minimizing the deviational variables and vector of deviational variables, respectively, for goal achievement, and

$$\sum_{k=1}^{K}(w^-_{kL} + w^+_{kU}) = 1 \text{ and } \sum_{r=K+1}^{2K-1}(w^-_{rL} + w^+_{rU}) = I, \quad (11)$$

where 'I' is a column vector with all entries equal to 1.

Then, the executable GP model of the problem can be presented as:

Find $X(X_1, X_2, \ldots, X_K)$ so as to:

$$\text{minimize}\, Z = \lambda\{\sum_{k=1}^{K}(w^-_{kL}d^-_{kL} + w^+_{kU}d^+_{kU}) + \sum_{r=K+1}^{2K-1}(w^-_{rL}d^-_{rL} + w^+_{rU}d^+_{rU})\} + (1-\lambda)D,\ 0 < \lambda < 1$$

and satisfy the goal expressions in (8) and (9), subject

$(d^-_{mL} + d^+_{mU}) - D \leq 0, m = 1,2,\ldots,2K-1$

and the given system constraints in (2).
(12)

Now, for the developed GP model of the proposed problem, the task of all the DMs is to search the solution to satisfy the goal levels to the extent possible by evaluating the defined regret function for the overall benefit of the organization.

5.2. GA for IvP model

Now, since GA is a goal satisficer rather than optimizer, the proposed GA scheme can be employed here to minimize the regret function 'Z' and thereby to reach a satisfactory decision for proper distribution of decision powers to the DMs.

Here, the fitness function appears as:

$$\text{eval}(E_v) = (Z)_v = [\lambda\{\sum_{k=1}^{K}(w^-_{kL}d^-_{kL} + w^+_{kU}d^+_{kU}) + \sum_{r=K+1}^{2K-1}(w^-_{rL}d^-_{rL} + w^+_{rU}d^+_{rU})\} + (1-\lambda)D]_v \quad (13)$$

where, v represents a chromosome.

Here, the best chromosome E^* with highest fitness value at each generation is determined as:

$$E^* = \min\{\text{eval}(E_v) \mid v = 1,2,\ldots,\text{pop\_size}\}$$

6. An illustrative example

To illustrate the proposed approach, the following trilevel programming problem with interval coefficients is solved to expound the model.

Find $X(x_1, x_2, x_3)$ so as to

$\max_{x_1}\text{imize}\; Z_1(x_1, x_2, x_3) = [5,7]x_1 + [1,3]x_2 + [2,4]x_3$

(top-level problem)

where, for given $x_1; x_2, x_3$ solve

$\max_{x_2}\text{imize}\; Z_2(x_1, x_2, x_3) = [2,4]x_1 + [4,6]x_2 + [3,5]x_3$

(middle-level problem)

where, for given x_1 and $x_2; x_3$ solves

$\max_{x_3}\text{imize}\; Z_3(x_1, x_2, x_3) = [1,3]x_1 + [6,8]x_2 + [1,2]x_3$,

(bottom-level problem)

subject to

$$x_1 + x_2 + x_3 \geq 4.80$$
$$5x_1 + x_2 + 6x_3 \leq 11.85$$
$$2x_1 + 3x_2 + x_3 \leq 13.60$$
$$x_1, x_2, x_3 \geq 0$$
(14)

Now, to solve the problem by employing the proposed GA scheme, the following genetic parameters which are widely used as efficient ones in conventional GA methods are adopted in the solution search process.

Population Type	: Bit string
Population Size	: 20
Selection	: Roulette
Crossover Function	: Single Point
Crossover Fraction	: 0.8
Mutation Rate	: 0.08
Number of generations	: 100

The GA is implemented using Optimization Toolbox in MATLAB (Version 7.10.0.499 (R010a)). The execution is made in an Intel Pentium IV processor with 2.66 GHz. clock-pulses and 1 GB RAM.

Now, following the procedure and employing the GA scheme, the individual best and least solutions, respectively, of the objectives of the DMs of the successive hierarchical levels are obtained as:

$$(x_1^{1b}, x_2^{1b}, x_3^{1b}; T_1^{*b}) = (1.6437, 3.4261, 0.0343;\ 21.9210),$$
$$(x_1^{1l}, x_2^{1l}, x_3^{1l}; T_1^{*l}) = (-0.4768, 3.6023, 0.3106; 1.8394)$$

(top level DM's solution)

$$(x_1^{2b}, x_2^{2b}, x_3^{2b}; T_2^{*b}) = (0, 4.1029, 1.2912;\ 31.0735),$$
$$(x_1^{2l}, x_2^{2l}, x_3^{2l}; T_2^{*l}) = (1.1602, 2.6272, -0.4628;\ 11.4410)$$

(middle-level DM's solution)

$$(x_1^{3b}, x_2^{3b}, x_3^{3b}; T_3^{*b}) = (0, 4.3551, 0.5347; 35.9102)$$
$$(x_1^{3l}, x_2^{3l}, x_3^{3l}; T_3^{*l}) = (1.1602, 2.6272, -0.4628; 16.4608)$$

. (bottom-level DM's solution)

Then, the objectives of the DMs with the target intervals of the successive hierarchical levels can be set as in (6) using the concept of (5):

$$[5,7]x_1 + [1,3]x_2 + [2,4]x_3 = [2.00, 19.00]\ ,$$
$$[2,4]x_1 + [4,6]x_2 + [3,5]x_3 = [19.21, 30.00]\ ,$$
$$[1,3]x_1 + [6,8]x_2 + [1,2]x_3 = [18.00, 34.38]\text{, respectively.} \quad (15)$$

Now, in the decision situation, the top-level and the middle-level DMs give a relaxation on the values of x_1 up to 0.30 and that on x_2 up to 3.75, respectively, for the benefit of each of the successive lower level DMs. As such,

$$x_1^p = 0.30 (x_1^{1l} < 0.30 < x_1^{1b}) \text{ and}$$
$$x_2^p = 3.75 (x_2^{2l} < 3.75 < x_2^{2b})$$

are considered as the lower tolerance limits of the decisions x_1 and x_2, respectively.

Then, the control variables x_1 and x_2 of the top-level and middle-level DMs with the target intervals can be obtained as:

$$[1,1]x_1 = [0.3000, 1.6437]$$

and $[1,1]x_2 = [3.7500, 4.1029]$, respectively.

Now, the goals in conventional form are obtained as:

$$7x_1 + 3x_2 + 4x_3 + d_{1L}^- - d_{1L}^+ = 2$$
$$5x_1 + x_2 + 2x_3 + d_{1U}^- - d_{1U}^+ = 19$$
$$4x_1 + 6x_2 + 5x_3 + d_{2L}^- - d_{2L}^+ = 19.21$$
$$2x_1 + 4x_2 + 3x_3 + d_{2U}^- - d_{2U}^+ = 30$$
$$3x_1 + 8x_2 + 2x_3 + d_{3L}^- - d_{3L}^+ = 18$$
$$x_1 + 6x_2 + x_3 + d_{3U}^- - d_{3U}^+ = 34.3$$
$$x_1 + d_{4L}^- - d_{4L}^+ = 0.30$$
$$x_1 + d_{4U}^- - d_{4U}^+ = 1.6437$$
$$x_2 + d_{5L}^- - d_{5L}^+ = 3.75$$
$$x_2 + d_{5U}^- - d_{5U}^+ = 4.1029\ ,$$

where, $d_{kL}^-, d_{kL}^+, d_{kU}^-, d_{kU}^+ \geq 0$ with $d_{kL}^- . d_{kL}^+ = 0$; k=1,2,…,5.

(16)

The executable GP model can be obtained as:

Find $X(x_1, x_2, x_3)$ so as to

$$\text{minimize } Z = \lambda \sum_{k=1}^{5} (w_{kL}^- d_{kL}^- + w_{kU}^+ d_{kU}^+) + (1-\lambda) D$$

and satisfy the goal expressions in (16), subject to

$$(d_{1L}^- + d_{1U}^+) - D \leq 0,\ (d_{2L}^- + d_{2U}^+) - D \leq 0,$$
$$(d_{3L}^- + d_{3U}^+) - D \leq 0,\ (d_{4L}^- + d_{4U}^+) - D \leq 0,$$
$$(d_{5L}^- + d_{5U}^+) - D \leq 0,$$

and the system constraints in (14). (17)

Now, for simplicity and without loss of generality, introducing the equal weights $w_{kL}^- = w_{kU}^+ = \frac{1}{10}, k = 1,...,5$ for achievement of the goals and taking $\lambda = 0.5$, the problem is solved by employing the GA scheme with the consideration of the evaluation function defined in (13) as the fitness function.

The optimal decision out of 30 runs is obtained as: $(x_1, x_2, x_3) = (0.5654,\ 3.8701,\ 0.8588)$

The objective function values in the interval form are obtained as:

$Z_1 = [8.4\ , 19.0\ ],\ Z_2 = [19.2,\ 29.8\ ], Z_3 = [\ 24.6\ , 34.4\ ]$

The obtained interval-valued forms of the objectives are found within the target intervals. The result shows that the GA based EGP model in (17) gives a satisfactory decision from the view point of distributing the proper decision powers to the DMs in the decision making context.

Note1: If $\lambda = 0$ in (17), the model is transformed to MGP approach. The solution of the problem using proposed GA is $(x_1, x_2, x_3) = (0.3,\ 3.75,\ 0.75)$

The objective function values in the interval-valued form are obtained as:

$Z_1 = [6.8, 16.4], Z_2 = [17.8, 27.4], Z_3 = [23.6, 32.4]$.

Note2: If $\lambda = 1$ in (17), the model is transformed to WGP approach. The solution of the problem using proposed GA is

$(x_1, x_2, x_3) = (0.3,\ 3.75,\ 0.75)$

The result is same as that obtained by MGP method.

The objective function values in the interval-valued form are obtained as:

$Z_1 = [6.8, 16.4], Z_2 = [17.8, 27.4], Z_3 = [23.6, 32.4]$.

The results indicate that the proposed GA based approach offers a superior solution in terms of greater satisfaction of the DMs in the hierarchical decision making context.

The achievement of the objective values within their specified intervals under the three different approaches is shown in the Figure 1.

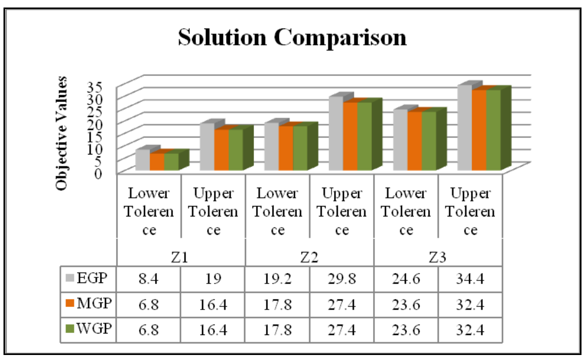

Figure 1: Achievement of Objective Values

A comparison shows that the proposed GA based EGP model is superior over the use of the conventional methodology of MGP and WGP from the point of view of proper distribution of decision powers to the DMs as well as arriving at the most satisfactory decision in the decision making environment.

7. Conclusions

In the proposed decision making environment of MLPP, the main advantage of using the GP formulations of IvP approach is that the difficulty of assigning the fixed parameter values to the problem does not arise here as in the case of conventional approaches. Further, GA as a goal satisficer is more fruitful to apply here to make a balance of the decision powers of the DMs in the MODM context. The proposed GA based EGP approach can also be extended for solving hierarchical decision problems having the characteristics of fractional programming, which is a problem for future study.

However, it is hoped that the approach presented here may open up many new vistas of future works in the field of large-scale hierarchical decentralized decision problems.

References

[1] M. Sakawa, I. Nishizaki, "Cooperative and Non-cooperative Multi-Level Programming", Springer, New York, 2009.

[2] R.M. Burton, "The Multilevel Approach to Organizational Issues of the Firm- A Critical Review" Omega, 5(4), 395-414, 1977.

[3] G. Anandaligam, "A mathematical programming model of decentralized multi-level system" Journal of the Operational Research Society, 39(11), 1021-1033, 1988.

[4] J.F. Bard, "An algorithm for solving the bi-level programming problem" Mathematics of Operation Research, 8(2), 260-270, 1983.

[5] O. Ben-ayed, "Bilevel Linear Programming" Computers and Operations Research, 20, 485–501, 1993.

[6] W. Candler, R. Townsley, "A linear two level programming problem" Computers and Operations Research, 9(1), 59-76, 1982.

[7] J. Fortunity-amat, and B. Mccarl, "A Representation and Economic Interpretation of a Two-level Programming Problem" Journal of the Operational Research Society, 32(9), 782-793, 1981.

[8] J.F. Bard, "Coordination of a multidivisional firm through two levels of management" Omega, 11(5), 457-465, 1983.

[9] M. Fampa, L.A. Barroso, D. Candal, L. Simonetti, "Bilevel Optimization Applied to Strategic Pricing in Competitive Electricity Markets" Computational Optimization and Applications, 39(2), 121-142, 2008.

[10] D. Cao, M. Chen, "Capacitated Plant Selection in a Decentralized Manufacturing Environment: A Bilevel Optimization Approach" European Journal of Operational Research, 169, 97–110, 2006.

[11] G. Anandaligam, "Multi-level programming and conflict resolution" European Journal of Operational Research, 51(2), 233-247, 1991.

[12] H.-J. Zimmermann, "Fuzzy Sets, Decision Making and Expert Systems", Kluwer Academic Publisher, Boston, Dordrecht, Lancaster, 1987.

[13] L.A. Zadeh, "Fuzzy Sets" Information and Control, 8(3), 338-353, 1965.
[14] R.H. Mohamed, "The Relationship between Goal Programming and Fuzzy Programming" Fuzzy Sets and Systems, 89, 215–222, 1997.
[15] C. Romero, "Handbook of Critical Issues in Goal Programming" Pergamon Press: Oxford, 1991.
[16] H.A. Simon, "Administrative Behavior", Fress Press, New York, 1957.
[17] A., Biswas, B.B. Pal, "A Fuzzy Multilevel Programming Method for Hierarchical Decision Making" Lecture Notes in Computer Science 3316, Springer, Berlin, 904-911, 2004.
[18] A.L. Soyster, "Inexact Linear Programming with Generalized Resource Sets" European Journal of Operational Research, 3, 217–237, 1979.
[19] R.E. Moore, "Interval Analysis" Prentice-Hall, New Jersey, 1966.
[20] G.R. Bitran, "Linear Multiobjective Problems with Interval Coefficients" Management Science, 26, 694–706, 1980.
[21] M. Inuiguchi, Y. Kume, "Goal Programming Problems with Interval Coefficients and Target Intervals" European Journal of Operational Research, 52, 345–361, 1991.
[22] M. Inuiguchi, M. Sakawa, "Minimax Regret Solution to Linear Programming with an Interval Objective Function" European Journal of Operational Research, 86, 526–536, 1995.
[23] C. Oliveira, C.H. Antunes, "Multiple Objective Linear Programming Models with Interval Coefficients – An Illustrated Overview" European Journal of Operational Research, 118, 1434–1463, 2007.
[24] B.B. Pal, S. Sen, "A Linear Goal Programming Approach to Multiobjective Fractional Programming with Interval Parameter Sets", International journal of mathematics in operational research, 3(6), 697-714, 2011.
[25] J.S.H. Kornbluth, R.E. Steuer, "Multiple objective linear fractional programming" Management Science, 27, 1024–1039, 1981.
[26] J.H. Holland, "Genetic Algorithms and Optimal Allocation of Trials" SIAM Journal of Computing, 2, 88-105, 1973.
[27] D.E. Goldberg, "Genetic Algorithms in Search, Optimization and Machine Learning" Addison-Wesley, Reading. MA, 1989.
[28] Z. Michalewicz, "Genetic Algorithms + Data Structures = Evolution Programs", 3rd edition. Spriger-Verlag, Berlin Heidelberg, New York, 1996.
[29] K. Deb, "Multi-objective Optimization using Evolutionary Algorithms", John Wiley & Sons Ltd, 2002.
[30] M. Gen, K. Ida, J. Lee, J. Kim, "Fuzzy Nonlinear Goal Programming Using Genetic Algorithm" Computers and Industrial Engineering, 33 (1 - 2), 39-42, 1997.
[31] B.B. Pal, D. Chakraborti, P. Biswas and A. Mukhopadhyay, An Application of Genetic Algorithm Method for Solving Patrol Manpower Deployment problems through Fuzzy Goal Programming in Traffic Management System: A Case Study" International Journal of Bio-Inspired Computation, 4(1), 47–60, 2012.
[32] R., Mathieu, L. Pittard, G. Anandaligam, "Genetic Algorithm Based Approach to Bilevel Linear Programming" Operations Research, 28(1), 1-21, 1994.
[33] M. Sakawa, I. Nishizaki, M. Hitaka, "Interactive Fuzzy Programming for Multi-level 0-1 Programming Problems with Fuzzy Parameters through Genetic Algorithms" Fuzzy Sets and Systems, 117(1), 95—111, 2001.
[34] B.B. Pal, D. Chakraborti, P. Biswas, "Using Genetic Algorithm for Solving linear Multilevel Programming Problems via Fuzzy Goal Programming" Communications in Computer and Information Science 140, Springer Verlag, 79-88, 2011.
[35] B.B. Pal, P. Biswas, D. Chakraborti, A. Mukhopadhyay, "A Genetic Algorithm Based Mixed 0-1 Goal Programming Approach to Interval-valued multiobjective Bilevel Programming Problem" International Journal of Fuzzy systems and Rough systems, 4(1), 25-32, 2011.
[36] J.P. Ignizio, "Goal Programming and Extensions", Lexington, Massachusetts, D. C. Health, 1976.
[37] C. Romero, "A General Structure of Achievement Function for a Goal Programming Model", European Journal of Operational Research, 153, 675 – 686, 2004.
[38] C. Romero, B. Vitoriano, "Extended Interval Goal Programming", Journal of the Operational Research Society, 50, 1280–1283, 1999.
[39] R.B. Flavell, A New Goal Programming Formulation. Omega, 4(6), 731-732, 1976.
[40] C. Romero, M. Tamiz, D.F. Jones, "Goal Programming, Compromise Programming and Reference Point Method Formulations: Linkages and Utility Interpretations" Journal of the Operational Research Society, 49(9), 986 – 991, 1998.

Papun Biwas is an Assistant Professor in the Department of Electrical Engineering, JIS College of Engineering, Kalyani, West Bengal, India. He received the M.Tech Degree in Electrical Engineering from the University of Calcutta in 2007. Currently he is a Research Fellow under the supervision of Dr. Anirban Mukhopadhyay and Professor Bijay Baran Pal, in the Department of Computer Science & Engineering, University of Kalyani, India. His research interests are soft Computing and multiobjective optimization.

Bijay Baran Pal is a Professor in the Department of Mathematics, University of Kalyani, India. He received his MSc in Mathematics, University of Kalyani in 1979, and then DIIT in computational Mathematics and Computer programming from Indian Institute of Technology (IIT), Kharagpur, India in 1980.
He was awarded the PhD by the University of Kalyani in 1988. He has published a number of research articles in different national and international journals including Elsevier Science. He has provided reviewer services to different international journals and international conferences. He is an editorial board member of the journals: International Journal of Applied Management Science and International Journal of Data Analysis Techniques and Strategies, Inderscience.com. His research interests cover different areas of soft-computing, pervasive computing, and multiobjective decision making (MODM) in inexact decision environment.

Anirban Mukhopadhyay is currently an Associate Professor and Head of the Department of Computer Science and Engineering, University of Kalyani, Kalyani, West Bengal. He did his B.E. from National Institute of Technology, Durgapur, India, in 2002 and M.E. from Jadavpur University, Kolkata, India, in 2004, respectively, both in Computer Science and Engineering. He obtained his Ph.D. in Computer Science from Jadavpur University in 2009. Dr. Mukhopadhyay is the recipient of the University Gold Medal and Amitava Dey Memorial Gold Medal from Jadavpur University in 2004 for ranking first class first in M.E. He also received Erasmus Mundus fellowship in 2009 to carry out post-doctoral research at University of Heidelberg and German Cancer Research Center (DKFZ), Heidelberg, Germany during 2009-10. Dr. Mukhopadhyay also visited I3S laboratory, University of Nice Sophia-Antipolis, Nice, France in 2011 as a Visiting Professor. He has co-authored one book and about 80 research papers in various International Journals and Conferences. Dr. Mukhopadhyay attended in many national and international conferences in India and abroad. He is a senior member of of Institute of Electrical and Electronics Engineers (IEEE), USA, and member of Assciation for Computing Machinery (ACM), USA, and International Association of Engineers (IAENG), Hong Kong. Dr. Mukhopadhyay is an active member of the board of reviewers in various International Journals and Conferences. His research interests include soft and evolutionary computing, data mining, multiobjective optimization, pattern recognition, bioinformatics, and optical networks.

Debjani Chakraborti is an Assistant Professor in the Department of Mathematics, Narula Institute of Technology, Agarpara, Kolkata (India). She received the M. Sc. Degree in Applied Mathematics from the University of Kalyani (India) in 1999, and then M.Phil Degree in Applied Mathemtics from University of Calcutta (India) in 2006. Currently she is a Research Fellow under the supervision of Professor Bijay Baran Pal, in the Department of Mathematics, University of Kalyani, India. Her research interests are soft Computing and multiobjective decision making in inexact environment in the field of Operations Research.