

## Constant Modulus Blind Equalization for Time-Varying MIMO-FIR Channels with Pulse Estimation

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### Abstract

*The constant modulus (CM) property of source signal can be mostly utilized to equalize the single-input multiple-output and finite impulse response (FIR) channels. In existing, the channels are blindly equalized and the equalization methods are based on higher order statistics that estimates all non-zero channel pulsation under time-invariance assumption. However, the assumption cannot be satisfied time varying multiple input multiple output (MIMO) applications ex. Mobile communication. In this paper, the proposed algorithm which extends the CM criterion to blind equalization using complex exponential basic expansion model (CE-BEM) and the channel is assumed as time varying MIMO-FIR. The method only employ the Second order statistics (SOS) and finally, it estimates only one pulsation. In this way, the system increases the SNR of the transmitted symbols and produces most beneficial result in time-varying channels. The fast convergence is also achieved through zero forcing equalization.*

### Keywords

*Blind equalization, CE-BEM, constant modulus criterion, MIMO, second order statistics, zero forcing equalization.*

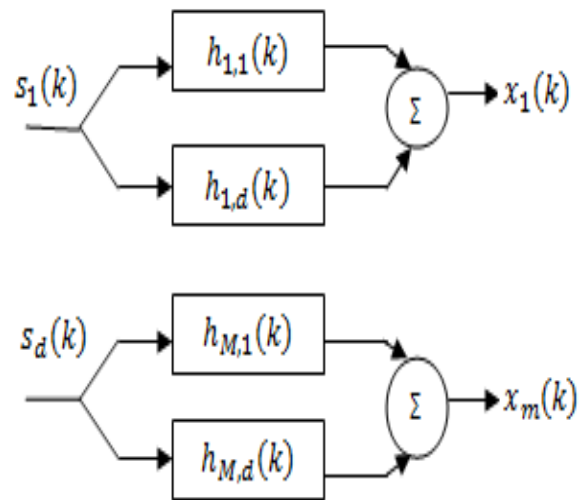
### 1. Introduction

Blind channel equalization, an approach commonly used to counter the effects of channel distortion, can be viewed as the application of filter (i.e. equalizer) to the received signal. The equalizer attempts to extract the transmitted symbol sequence by counteracting the effects of ISI, thereby improving the probability of correct symbol detection. However, blind techniques for linear time-invariant (TI) systems have found widespread applications in time series modeling, econometrics, exploration seismology, and equalization of communication channels, just to name a few.

But, Now-a-days most of the communication systems are fast time varying ex. Mobile communication. The Inter symbol Interference (ISI) and channel noise

may be a common problem faced in wireless and mobile communication systems. Traditionally, the conventional equalization technique proposed for communication systems to overcome the ISI degradation. The drawback is training sequence which makes this equalization inefficient in terms of lengthy training sequence. When wireless channels, especially mobile channels are fast time variant, training signals must be sent frequently. As such, a lot of bandwidth has to be occupied. For example, according to 900MHz GSM standard, 26-bits out of every 148-bit frames are used as training signals. Whereas the blind equalization do not need any reference signals due to the transmission period.

In existing method, the constant modulus properties of source signals are blindly equalize the time invariant single input multiple output signals are proposed. Recently, Dezhong Peng and Yong Xiang proposed a novel method to estimate the basis frequencies for blind identification and equalization of time-varying single-input multiple-output (SIMO) [1] FIR channels.



**Fig.1: Example for multiple input multiple output channels**

By using BEM approximation of time-varying channel, the time-invariant coefficients can be estimated. In the proposed work, the CE-BEM has

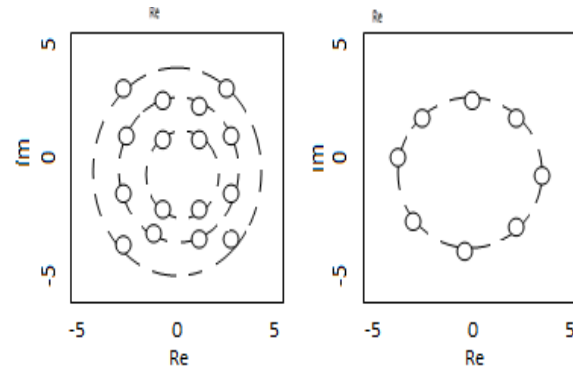
been used to describe the TV FIR channels [5]–[9]. The time-varying impulse response of rapidly fading mobile communication channels is expanded over a basis of complex exponentials that arise due to Doppler effects encountered with multi-path propagation. This model is particularly useful when the receivers are constantly moving and the transmitters and a few dominant reflectors are stationary [5], [8], [9]. It is justified in [5] and [6] that the CE-BEM is effective in tracking the variations in land mobile channels.

The constant modulus criterion [1]–[3] ranks among the most widely employed methods for blind signal restoration. The most common implementation, in terms of a stochastic gradient algorithm, often requires hundreds or even thousands of iterations to converge, depending on channel conditions, equalizer length, signal to noise ratio, etc.,. The success of a stochastic gradient descent equalization is dependent on a certain amount of stationarity in the received process. Thus, throughout the paper, stationary source, noise processes and channels with slowly time varying impulse response is focused. As well; this criterion restores the constant envelope property of the signal and increase the SNR.

Several basis frequency estimation methods have been proposed in [9] and [12]. The adaptive maximum-likelihood estimation method in [9] estimates basis frequencies via a gradient search procedure with respect a non-quadratic cost function and thus suffers from local minima. Also, the computation of gradients requires knowledge of the channel coefficients, which is usually unknown in practical applications. The fourth-order statistics-based estimation method in [12] requires  $E[s^4(k)] \neq 0$ , where  $s(k)$  is the source signal and  $E[\cdot]$  denotes the mathematical expectation operator. Clearly, the inequality does not hold for many practical communication signals such as 8-PSK and 16-PSK signals. Hence, the frequency estimation method in [12] is unable to deal with some classes of signals with  $E[s^4(k)] = 0$  e.g., 8-PSK signal [9]. Apart from that, higher-order statistics (HOS) based methods need to use a very large number of samples to achieve reasonable statistical performance and are expensive in computation. The proposed method exploits only the second-order statistics of channel outputs and does not require strong conditions on the source signal.

In general, the constant modulus algorithm (CMA) seeks to minimize a cost defined by the CM criterion.

The CM criterion penalizes deviations in the modulus (i.e., magnitude) of the equalized signal away from a fixed value. In certain ideal conditions, minimizing the CM cost can be shown to result in perfect (zero-forcing) equalization of the received signal. Remarkably, the CM criterion can successfully equalize signals characterized by source alphabets not possessing a constant modulus [e.g., 16-quadrature amplitude modulation (QAM)], as well as those possessing a constant modulus (e.g., 8-PSK) (see Fig. 2). This paper shows, the zero-forcing (ZF) equalization can be achieved using the CM criterion and the result is accurate estimation of single pulsation for blind equalization of TV-MIMO FIR channels rather than all pulsation signals.



**Fig.2: 16-QAM without CM & 8-PSK with CM**

## 2. Proposed Channel Model

Consider a TV MIMO-FIR channel system with  $M$  inputs and  $M$  outputs. In this system, the output is given by

$$X_q(k) = \sum_{l=0}^L \sum_{q=0}^{Q-1} h_q(k; l) S_q(k-l) + v_q(k) \quad (1)$$

Where  $x_q(k) = [x_1(k), x_2(k) \dots \dots x_M(k)]^T$  denotes the output vector and after stacking some successive observations of the channel outputs which yields  $\bar{X}(k) = [X^T(k), X^T(k-1), \dots \dots X^T(k-K)]^T \quad (2)$   $s_q(k) = [s_1(k), s_2(k), \dots \dots s_M(k)]^T$ , is the source signal with CM constellation,  $v_q(k) = [v_1(k), v_2(k), \dots \dots v_q(k)]^T$ , is the vector with  $M$  additive white Gaussian noise signals, independent of  $s(k)$ ,  $h_q(k; l) = [h_1(k; l), h_2(k; l), \dots \dots h_q(k; l)]^T$  is the time varying impulse response vector, and the subscript  $T$  stands for the transpose operation. In this paper, we propose the CE-BEM under the assumption of path delay vary with time linearly and

the time varying channel impulse response  $h_q(k; l)$  in the CE-BEM are described as the finite linear combination of some complex exponential basis functions, say  $Q$ , i.e.  $\{e^{j\omega_0 k}, e^{j\omega_1 k}, \dots, e^{j\omega_{Q-1} k}\}$  where  $k$  is the discrete time index and  $j = \sqrt{-1}$ .

$$h_p(k; l) = \sum_{q=0}^{Q-1} h_{p,q}(l) e^{j\omega_q k} \quad l = 0, 1, \dots, L$$

$$p = 1, 2, \dots, P \quad (3)$$

Where  $h_{p,q}(l)$  are some time-invariant coefficients and  $\omega_1, \omega_2, \dots, \omega_Q$  are distinct real numbers called pulsations (or basis frequencies). In (2), the channel variations are caused by the complex exponential basis function  $\{e^{j\omega_q k}\}_{q=0}^{Q-1}$ , which are solely determined by the  $Q (Q \geq 2)$  distinct basis frequencies  $\{\omega_0, \omega_1, \dots, \omega_Q\}$  with  $\omega_0 < \omega_1 < \dots < \omega_{Q-1}$ .

Use 1 & 2, it follows that

$$x_q(k) = \sum_{l=0}^L \sum_{q=0}^{Q-1} e^{j\omega_q k} h_{p,q}(k; l) s_q(k-l) + v_q(k) \quad (4)$$

Clearly the equation shows that, the recognition of time varying channels is equivalent to the estimation of the basis frequencies  $\{\omega_q\}_{q=0}^{Q-1}$  as well as the expansion coefficients  $h_{p,q}(l)$ .

To proceed, we define the  $M \times Q$  matrices  $H_n (n = 0, 1, \dots, L)$ , the  $M(K+1) \times Q(P+1)$  matrix, and the  $Q(P+1) \times Q(P+1)$  diagonal matrix  $C(k)$  as follows:

$$H_1 = \begin{bmatrix} e^{j\omega_1 l} h_{1,1}(l) & e^{j\omega_2 l} h_{1,2}(l) & \dots & e^{j\omega_Q l} h_{1,Q}(l) \\ e^{j\omega_1 l} h_{2,1}(l) & e^{j\omega_2 l} h_{2,2}(l) & \dots & e^{j\omega_Q l} h_{2,Q}(l) \\ \vdots & \vdots & & \vdots \\ e^{j\omega_1 l} h_{M,1}(l) & e^{j\omega_2 l} h_{M,2}(l) & \dots & e^{j\omega_Q l} h_{M,Q}(l) \end{bmatrix} \quad (5)$$

And

$$H = \begin{bmatrix} H_0 & \dots & H_L & \dots & 0_{M \times Q} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_{M \times Q} & \dots & H_0 & \dots & H_L \end{bmatrix}$$

$$C(k) = \text{diag} \left( \underbrace{e^{j\omega_1 k}, \dots, e^{j\omega_Q k}}_Q, \dots, \underbrace{e^{j\omega_1(k-P)}, \dots, e^{j\omega_Q(k-P)}}_Q \right) \quad (6)$$

Where  $P = K + L$ . The  $H$  is referring to toeplitz matrix and  $c(k)$  is a diagonal matrix. Aforementioned equations, we summarize the  $\bar{X}(k)$  as follows that

$$\bar{X}(k) = HC(k)\bar{s}(k) \quad (7)$$

The task here is to estimate the basic frequencies from second order statistics of the channel

outputs  $x_1(k), x_2(k) \dots, x_q(k)$ . To proceed, we assume in the sequel that

A1) The source signal  $s_q(k)$  is a zero-mean white stationary stochastic process and has  $S = 2^R$  constellation points, which is uniformly distributed on the unit circle, where  $R \geq 2$  is a positive integer.

A2) The noise sequences  $v_q(k) = [v_1(k), v_2(k), \dots, v_q(k)]$  are stationary, temporally and spatially white, and independent of  $s_q(k)$ .

A3) The minimum basis frequency  $\omega_0 = 0$  and the maximum basis frequency is  $\omega_Q$ . i.e.,  $0 = \omega_0 < \omega_1 < \dots < \omega_Q$ .

### 3. Blind Equalization Using the Constant Modulus Criterion

Let  $g$  be an equalization vector and define

$$y(k) = g^T \bar{x}(k) \quad (8)$$

The zero-forcing equalizer can be developed with the equalization vector. i.e.,  $\hat{g}$  satisfies the following condition

$$\hat{g}H = [0, \dots, 0, \zeta, 0, \dots, 0] \quad \text{and}$$

$$\zeta = 1 \text{ (Perfect equalization)}$$

To find the preferred equalization vector  $\hat{g}$ , and show the CM criterion as follows,

$$J(g) = \frac{1}{4} E \{ [|y(k)|^2 - 1]^2 \} \quad (9)$$

Generally, the stochastic gradient algorithm minimizes the CM cost function and has been developed as

$$g(k+1) = g(k) - \mu (|y(k)|^2 - 1) y(k) \bar{x}^*(k) \quad (10)$$

Where  $\mu$  is the step size and the superscript \* denotes the complex conjugate function. By proper tuning of the step size parameter, convergence usually occurs within a few iterations (between 5 and 10 iterations is typical). It has been established in [19] and [10].

Furthermore, we minimize the cost function by modifying the above algorithm as follows,

$$\min J = E [|g^H y(k)|^2 - 1] \quad (11)$$

The update rule is given by

$$g_{k+1} = g_k - \mu \cdot K [|g^H y(k)|^2 - 1] y(k) y^H(k) g_k \quad (12)$$

Here we assign the numerical constant  $K = 2$ . Then after we estimate the zero forcing equalization vector  $\hat{g}$ . From that we compute the output vector  $y(k)$ . i.e

$$y(k) = \hat{g}^T \bar{X}(k) \quad (13)$$

**Estimation of unknown pulsation:**

Furthermore we express the aforementioned equation (13) as

$$y(k) = \hat{g}^T HC(k)\bar{s}(k) = \zeta s(k - t_0) e^{j\omega_q(k-t_0)} \quad (14)$$

Where  $\omega_{\bar{q}} \in \{\omega_1, \omega_2, \dots, \omega_Q\}$ , and  $0 \leq t_0 \leq P$ . Now the remaining task is estimation of unknown pulsation  $\omega_{\bar{q}}$  and then determine the set  $\{\omega_{\bar{q}} - \omega_q\}_{q=1}^Q$  by using the Second-order statistics  $E[y(k) \cdot \sum_{t=0}^K x_p^*(k-t)]$ . Finally we recover the source signal  $s(k)$  by removing the  $e^{j\omega_{\bar{q}}(k)}$  from  $y(k)$ . According to the assumption (3)  $\omega_0 = 0$  and the largest element in the set is  $\omega_{\bar{q}} - \omega_0 = \omega_{\bar{q}}$  in theory and is estimate of  $\omega_{\bar{q}}$  in practice.

The summarization of CM based blind equalization algorithm is formulated as follows

- **Step1.** Generate the source signal  $s(k)$  which are uniformly distributed on the unit circle and then add white noise to that signal.
- **Step2.** Next Compute the equalizer output vector  $y(k) = \hat{g}^T \bar{X}(k)$
- **Step3.** Compute the Fourier series  $E[y(k) \cdot \sum_{t=0}^K x_p^*(k-t)]$  and then determine the set  $\{\omega_{\bar{q}} - \omega_q\}_{q=1}^Q$ .
- **Step4.** Find the largest element in  $\{\omega_{\bar{q}} - \omega_q\}_{q=1}^Q$  as the estimate of the frequency  $\omega_{\bar{q}}$ .
- **Step5.** Estimate the source signal  $s(k)$  from  $y(k) \cdot e^{-j\omega_{\bar{q}}k}$

Fig.4. shows the bit error rate performance (BER) of the proposed algorithm with the source of 8-PSK signal.

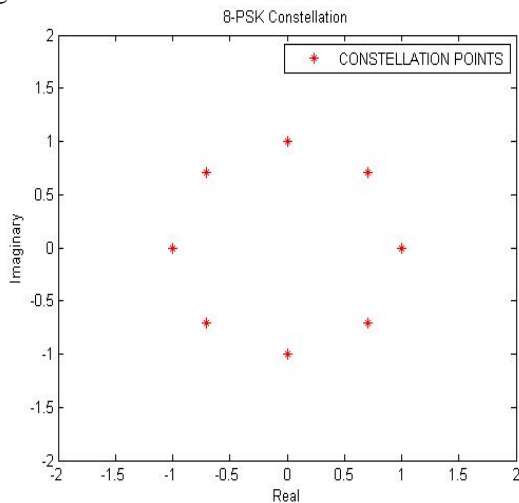


Fig.3: 8-PSK source signal

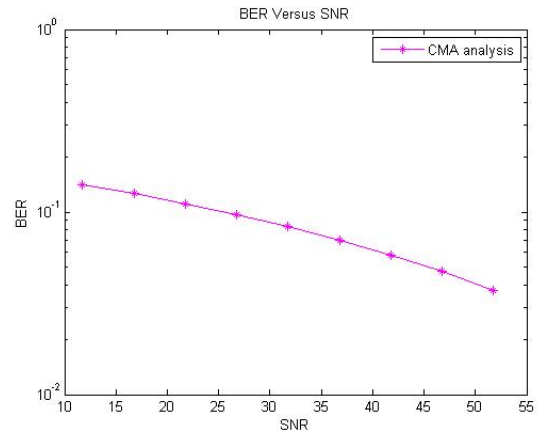


Fig.4: BER versus SNR analysis

**Numerical Simulations:**

**Example:**

Let us consider the TV  $2 \times 4$  MIMO FIR channel that has four outputs i.e  $L = 2$  and  $M = 4$ . The time invariant channel coefficients are generated randomly and we choose that coefficient as  $K = 1$ . Fig.1 shows 8-PSK signal, it is used as the source signals  $(k)$ . The blind equalization methods in (1), can works this signal with SIMO system but not in MIMO. Even most of the equalization algorithm restrict the PSK signals; refer [6] & [9]. The CM based algorithm can effectively deal the above mentioned signals. Here we assume 4 pulsations: 1)  $\omega_0 = 0$  ; 2)  $\omega_1 = 2\pi/200$ ; 3)  $\omega_2 = 2\pi/110$  ; 4)  $\omega_3 = 2\pi/40$ ;

Fig.5. Shows the bit error rate (BER) of the proposed algorithm versus the signal-to-noise ratio (SNR) under 3 channel scenarios 1) two pulsation 2) three pulsation 3) four pulsation.

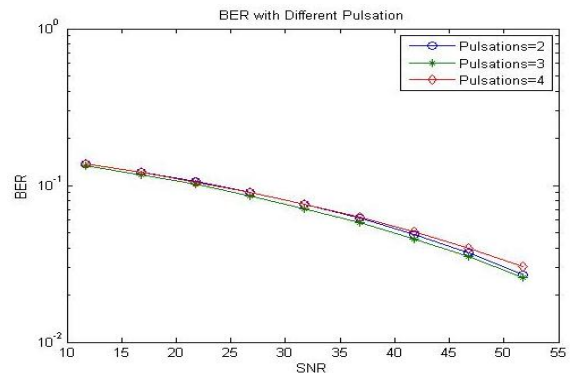
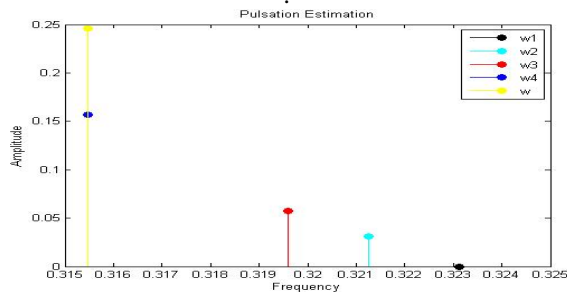


Fig.5: BER Vs SNR under three different scenarios

The unknown pulse  $\omega_{\bar{q}}$  is estimated from the set  $\{\omega_{\bar{q}} - \omega_q\}_{q=1}^Q$  by using the second order statistics. Fig.6. Shows the pulse estimation of the proposed CM based algorithm which determine the  $\omega_{\bar{q}}$ .



**Fig.6: Pulse Estimation**

## 4. Conclusion

In this paper, the feasibility of utilizing the CM based blind equalization for TV MIMO channels are investigated. The proposed algorithm equalizes the TV MIMO channels and finally recovers the source signal that is described by CE-BEM. As a result, it can be applied to wide ranges of applications. Furthermore, the algorithm uses only the second-order statistics instead of higher order moments. Therefore, the computation complexity could be substantially reduced. This approach results in a more accurate estimation outcome. Simulation examples illustrate the good equalization performance of the CM based algorithm to the existing approach.

## References

- [1] Zhang Yi, Senior Member, IEEE, and Shui Yu, Member, "CM-Based Blind Equalization of Time-Varying SIMO-FIR Channel With Single Pulsation Estimation" Dezhong Peng, Member, IEEE Yong Xiang, IEEE transactions on vehicular technology, vol. 60, no. 5, June 2011.
- [2] D. Peng, Y. Xiang, and D. Huang, "Estimation of basis frequencies for time-varying SIMO channels: A second-order method," IEEE Trans. Signal Process., vol. 58, no. 8, pp. 4026–4039, Aug. 2010.
- [3] A.Coskun and I. Kale, "Blind multidimensional matched filtering techniques for single-input-multiple-output communications," IEEE Trans. Instrum. Meas., vol. 59, no. 5, pp. 1056–1064, May 2010.
- [4] J. K. Tugnait and W. Luo, "Blind space-time multiuser channel estimation in time-varying DS-

- CDMA systems," IEEE Trans. Veh. Technol., vol. 55, no. 1, pp. 207–218, Jan. 2006.
- [5] J.K. Tugnait, L. Tong, and Z. Ding, "Single-user channel estimation and equalization," IEEE Signal Processing Magazine, vol. 17, no. 3, pp. 16–28, May 2000.
- [6] F.-J. Chen, S. Kwong, and C.-W. Kok, "Blind MMSE equalization of SISO IIR channels using oversampling and multichannel linear prediction," IEEE Trans. Veh. Technol., 2011, Digital Object Identifier: 10.1109/TVT.2007.905249.
- [7] M. K. Tsatsanis and G. B. Giannakis, "Modeling and equalization of rapidly fading channels," Int. J. Adapt. Control Signal Process., vol. 10, no. 2/3, pp. 159–176, Mar. 1996.
- [8] G. B. Giannakis and C. Tepedelenlioglu, "Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels," Proc. IEEE, vol. 86, no. 10, pp. 1969–1986, Oct. 1998.
- [9] H. Liu and G. B. Giannakis, "Deterministic approaches for blind equalization of time-varying channels with antenna arrays," IEEE Trans. Signal Process., vol. 46, no. 11, pp. 3003–3013, Nov. 1998.
- [10] C. Tepedelenlioglu and G. B. Giannakis, "Transmitter redundancy for blind estimation and equalization of time- and frequency-selective channels," IEEE Trans. Signal Process., vol. 48, no. 7, pp. 2029–2043, Jul. 2000.
- [11] E. Bai and Z. Ding, "Blind decision feedback equalization of time-varying channels with DPSK inputs," IEEE Trans. Signal Process., vol. 49, no. 7, pp. 1533–1542, Jul. 2001.
- [12] A. V. Dandawate and G. B. Giannakis, "Asymptotic theory of mixed time averages and Kth-order cyclic-moment and cumulant statistics," IEEE Trans. Inf. Theory, vol. 41, no. 1, pp. 216–232, Jan. 1995.
- [13] Y. Sato, "A method of self-recovering equalization for multilevel amplitude modulation systems," IEEE Trans. Commun., vol. COM-23, no. 6, pp. 679–682, Jun. 1975.



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