International Journal of Advanced Computer Research (ISSN (print): 2249-7277 ISSN (online): 2277-7970)
Volume-3 Number-3 Issue-11 September-2013

Optimal Coordination of Directional Overcurrent Relays using Interval Two Phase Simplex Linear Programming

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Abstract

The coordination of directional over current relays in interconnected power networks is formulated as an optimization problem. When two protective apparatus installed in series have characteristics, which provide a specified operating sequence, they are said to be coordinated or selective. The coordination of directional overcurrent relays pose serious problems in the modern complex power system networks, which are interconnected. In this paper, the coordination of directional overcurrent relays is formulated using Linear programming two phase simplex method and intervals. The objective function to be minimized is defined as sum of the time dial settings of all the relays. The inequality constraints guarantee the coordination margin for each primary/backup relay pair having a fault very close to relay pair. Using this formulation the size of optimization problem is greatly reduced. The proposed method is implemented to a simple radial and IEEE 3-bus systems and proves its ability for optimal setting of relays without any miscoordination and in minimum operating time with bounded values.

Keywords

Coordination, Interconnected networks, Linear programming, Two phases Simplex, Optimal coordination, Interval Analysis

1. Introduction

Overcurrent relays (OCRs) and Directional overcurrent relays (DOCRs) are widely used for the protection of radial and ring sub transmission systems, and distribution systems. Directional overcurrent relaying is used mainly for the primary protection of ring distribution systems, as the same magnitude of fault current can flow in either direction [1].They are also used for secondary protection in transmission systems [2]. Relays in different locations will detect greatly different currents during the same fault. The relay coordination problem is to determine the sequence of relay operations for each possible fault location so that faulted section is isolated to provide sufficient coordination margins without excessive time delay [3].The most vital task when installing directional relays on the system is selecting their suitable settings such that their fundamental protective function is met under the requirements of sensitivity, selectivity, reliability and speed [4]. The overcurrent relay coordination in ring fed distribution networks is a highly constrained optimization problem. The ultimate objective being improved power system reliability. The calculation of the time dial setting (TDS) and pick up current (Ipu) setting of the relays is the core of the coordination.

This paper discusses interval linear programming method to compute the quantities of interest with single interval computation, leading to the set of possible values, corresponding to allowable ranges for the constraints and initial data. In this paper, a new problem formulation for the coordination of directional overcurrent relays is proposed. A linear programming technique, two phase simplex method is used with intervals. The objective function is stated as a sum of operating times of all primary relays irrespective of the type and the location of the fault for maximum close-in faults, and constraints considered are based on maximum close-in faults as well.

The rest of the paper is organized as follows. In section 2 we give the optimal coordination of the relays where we give the problem formulation and the relay characteristics and settings. In section 3 we give an introduction of the interval analysis and in section 4 we give the algorithm of the two phase simplex method where interval analysis is used. In section 5 we give the results on the test problems followed by conclusion section.

2. Literature Review

In 2005, Birla et al [3] proposed that earlier, time consuming procedures performed for the coordination of directional relays involved tireless

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manual calculations. Basically, there are two methods used for finding proper coordination-conventional philosophy and the parameter optimization techniques. The basis of the conventional protection philosophy is the concept of pre determinism, i.e. analysis of all faults, abnormal operating conditions and system contingencies are predetermined. Drawback of these methods being inability to determine relay response for a condition not previously included in the analysis, since it is practically impossible to handle all the operating conditions of concern in advance. The optimization theory has simplified the coordination philosophy and because of inherent advantages, it is gaining popularity.

In 1988,Urdaneta et al [6, 7] assumed a middle line fault in the problem formulation, to reduce the problem dimensionality. The authors used nonlinear programming for determining the optimal settings of the pickup current and a linear programming (Simplex method) for optimizing the time dial settings of the relays subject to the coordination constraints. In 1996, Chattopadhyay et al [8] applied the linear programming technique only to minimize operating time while the pickup currents are selected based on experience. In [8], the objective function is a weighted sum of operating times of the primary relays for close-in faults, and since the lines are short and approximately of equal length, equal weights (=1) were assigned for the operating times of all the relays.

3. Optimal Coordination of Overcurrent Relays

In Directional OC relays there are two settings: time dial setting (*TDS*) and pickup current setting (I_{PU}). Directional relays allow for continuous time dial setting and discrete pickup current setting. The coordination problem is linear if pickup current is considered to be fixed. This is solved to calculate the optimal *TDS* of the relays [4]. In the coordination problem of DOCRs, the aim is to determine the time multiplier setting and pickup current setting of each relay, so that the overall operating time of the primary relays is minimized [5].

3.1 Problem statement

Optimal coordination of overcurrent relays in the power network is a problem that could be stated as an optimization problem and depends on considering a variable. The problem could be linear or nonlinear depending on formulation. The objective function of operating times of the primary relays is optimized subject to keeping the operation of the backup relays coordinated. One possible approach to achieve minimum shock to the system due to faults would-be to minimize a sum of the operating times of all primary relays hoping that the operating times of individual primary relays would be close to the minimum individual operating times that might be possible. The problem formulation can be demonstrated with the help of Figure 1 and by assuming network consisting of n relays, the objective function J to be minimized can be expressed as [6], [7] and [8]:

$$J = \sum \quad W_i T_{ii} (1)$$

$$subject\ to: T_{ji} \geq T_{ii} - CTI \qquad (2)$$

where $T_{ii}$is the operating time of the primary relay $R_i$for a close-in fault i,$T_{ji}$is the operating time of the backup relay $R_j$for the same close-in fault at i, and *CTI* is the coordination time interval. *W* is weight assigned for operating time of the relay R_i. In distribution system, since the lines are short and are of approximately equal length, equal weight i.e., W_i= 1 for i = 1,…, n,is assigned for operating times of all the relays[8].

3.2 Relay Characteristics and Settings

Any directional overcurrent relay is combination of an instantaneous unit which is time independent

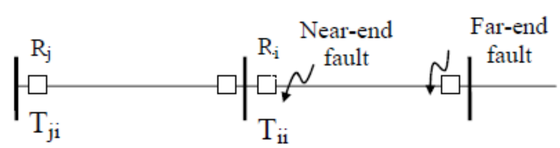

Figure 1: A power system showing primary and backup relays

and an inverse overcurrent unit which is time dependent. The time dependent unit has two values to be set, the pickup current, I_{PU} and the time dial setting, *TDS*. The preset value of current for which relay operates is the pickup value of the current. The time dial setting defines the operating time, *T* of the relay for each current value. The characteristics of the OCR are given as a curve of *T* versus *M*, where *M* is a multiple of the pickup current and is given by

$$M = \frac{I}{I_{PU}} (3)$$

and I is the relay current (overload/fault current). Here the overcurrent relay is conformed to the following IEC characteristic [9]. The inverse overcurrent relay characteristics can be approximated by the following expression.

$$T = k_1 \frac{TDS}{(M^{k2}-1)} \qquad (4)$$

where, k_1 and k_2 are constants that depend on the relay characteristics. The relation between the operating time, T of the time overcurrent unit and the multiple of pickup current M, is nonlinear [10]. The multiple of pickup current of the relays can be predetermined and so for a fixed M, the above equation can be rewritten as

$$T = \alpha \, TDS \qquad (5)$$

where,

$$\alpha = \frac{k1}{(M^{k2}-1)} (6)$$

By substituting (5) in (1), the objective function becomes

$$J = \sum_{i=1}^{n} \alpha_i \, TDS_i \qquad (7)$$

In equation (7), $\alpha_i's$ donot have any effect on the optimal solution and can be assumed 1. They are predetermined from (6) and substituted in (2). The values of TDS_i's are obtained by minimizing J given by (7) and satisfying the coordination between the primary and backup relays given by (2). The objective function is optimized using the well-known revised simplex linear programming technique and intervals [11], [12] subject to the condition that the operation of the backup relays remains properly coordinated.

4. Interval Arithmetic [13]

Interval arithmetic is an arithmetic defined on sets of intervals, rather than on sets of real numbers. It has been invented by R. E. Moore. The power of interval arithmetic lies in its implementation on computers. In particular, outwardly rounded interval arithmetic allows rigorous enclosures for the ranges of operations and functions. This makes a qualitative difference in scientific computations, since the results are now intervals in which the exact result must lie. It has been used recently for solving ordinary differential equations, linear systems, optimization, etc.

Let

$$\boldsymbol{x} = ([a,b] | a \leq b, a, b \in R)$$

be a real interval, where a is the infimum (lower endpoint) and b is the supremum (upper endpoint) of $\boldsymbol{x}$. The width of interval is defined as $w(\boldsymbol{x}) = a\text{-}b$. The midpoint of the interval is defined as $m(\boldsymbol{x}) = (a+b)/2$. For an n dimensional interval vector $\boldsymbol{x}^* = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n]$, the midpoint of interval vector $\boldsymbol{x}^*$ is given by $m(\boldsymbol{x}^*) = [m(\boldsymbol{x}_1), m(\boldsymbol{x}_2), \ldots, m(\boldsymbol{x}_n)]$. The width of interval vector is $w(\boldsymbol{x}^*) = [w(\boldsymbol{x}_1), w(\boldsymbol{x}_2), \ldots, w(\boldsymbol{x}_n)]$. A degenerate interval has both its lower and upper endpoints same. Let $\boldsymbol{x} = [a, b]$ and $\boldsymbol{y} = [c,d]$ be two intervals. Let +, -, *and / denote the operation of addition, subtraction, multiplication and division, respectively. If $\otimes$denotes any of these operations for the arithmetic of real numbers x and y, then the corresponding operation for arithmetic of interval numbers x and y is

$$\boldsymbol{x} \otimes \boldsymbol{y} = (x \otimes y | x \in \boldsymbol{x}, y \in \boldsymbol{y})$$

The above definition is equivalent to the following rules:

$$\boldsymbol{x} + \boldsymbol{y} = [a + c, b + d]$$
$$\boldsymbol{x} - \boldsymbol{y} = [a - d, b - c]$$
$$\boldsymbol{x} * \boldsymbol{y} = [\min(ac, bc, ad, bd), \max(ac, bc, ad, bd)]]$$
$$\frac{\boldsymbol{x}}{\boldsymbol{y}} = [a, b] * \left[\frac{1}{d}, \frac{1}{c}\right], \qquad if\ \ 0 \notin \boldsymbol{y}$$

An interval function $F(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ of intervals $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$ is an interval valued function of one or more variables. $F(x_1, x_2, \ldots, x_n)$ is said to be an interval extension of a real function $f(x_1, x_2, \ldots, x_n)$ iff$(x_1, x_2, \ldots, x_n) \in F(x_1, x_2, \ldots, x_n)$, whenever $x_i \in \boldsymbol{x_i}$for all i = 1,2……,n. F is said to be inclusion monotonic if

$$x_i \subset y_i \Rightarrow F(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}) \subset F(\boldsymbol{y_1}, \boldsymbol{y_2}, \ldots, \boldsymbol{y}_n)$$

Also$F(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})$contains the range of $f(x_1, x_2, \ldots, x_n)$.Interval functions $F(\boldsymbol{x})$ can be constructed in any programming language in which interval arithmetic is simulated or implemented via natural interval extensions. However, computing an interval bound carries a cost of 2 to 4 times as much effortas evaluating $f(x)$ [13], [14].

5. The Interval Two Phase Simplex Algorithm

The two phase Simplex method is used to find the non-negative value for the variables when the problem may have redundancies and/or inconsistencies and may not be solvable in non-negative numbers. Also, when the linear programming problem does not have slack variables for some of equations, two phase simplex solve the problem [12].
The constraints for relay settings are inequalities of ≥type. To convert these constraints into equality type, non-negative variable (surplus variable) is subtracted from left hand side. If surplus variables are taken as basics for initial solution, it will give infeasible solution as coefficient of surplus variable is -1.Thus, surplus variables does not lead to an initial feasible canonical form. So the Simplex technique in two part system is used.

5.1 Phase I

An artificial variable is defined for each surplus variable in same equation and a new objective function called an artificial cost function is introduced. It is defined as sum of all artificial variables introduced in the problem. In Phase I, artificial variables are reduced to zero using standard simplex procedure. When this is accomplished, Phase I is complete. If the artificial objective function depends only on the artificial variables and if its value is zero, it means artificial variables are non-basic variables. This implies that these variables were basic at the start of the procedure. When Phase I is completed both artificial objective function and variables are discarded from the table and Phase II begins.

- **Step 1**: Modify the constraints so that the right-hand side of each constraint is nonnegative. This requires that each constraint with a negative right-hand side be multiplied through by -1.
- **Step 2**: Identify each constraint that is now an = or ≥constraint and add an artificial variableto each of these constraints.
- **Step 3**: Convert each inequality constraint to standard form by adding a slack variable or an excess variable.
- **Step 4**: Define a quantity *w* as a new objective function given as the sum of all the artificial variables.
- **Step 5**: Declare all the variables and the coefficients of the original LPs objective function and the new objective function and constraints as intervals. All the coefficients and other entries of the matrix would be initially entered as degenerate intervals.
- **Step 6**: Solve an LP whose objective function now is min *w* = (sum of all the artificial variables) which will force the artificial variables to be zero.
- **Step 7**: The optimal value of *w* is equal to zero, and the resulting array will be in canonical form.
- **Step 8**: No artificial variables are in the optimal Phase I basis. We initiate Phase II by eliminating the *w* equation and drop all columns in the optimal Phase I tableau that corresponds to the artificial variables.

5.2 Phase II

It is standard Simplex technique [12], [11] applied to the problem after the end of Phase I. Apply the simplex algorithm to the new canonical form of system obtained from Phase I. An optimal bounded solution with the bounded minimum value of the objective function for the problem is obtained.

6. Results

In this paper, using the INTLAB toolbox [15], we develop the algorithm in MATLAB 6.1. We use here identical directional overcurrent relays with inverse characteristics for all the test examples. Initially a simple radial system and a 3-bus network taken from [16] and [7] respectively, are considered. We apply the developed methodology to these test systems. At first a simple radial system shown in Figure 2 is analyzed and the maximum fault currents at bus 1 and bus 2 are found to be 4000A and 3000A, respectively. The minimum operating time for the relays is considered as 0.2 second and CTI is taken as 0.57 second. The revised simplex method using intervals is used to calculate the values of TDS and the operating times of the primary and backup relays.

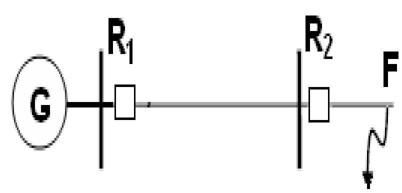

Figure 2: A radial system with fault

Table 1 gives the primary backup relay fault currents and Table 2 gives the CT ratios for the pickup current settings for the radial system.

Table 1: P/B relay pair for radial system

Backup relay	Fault current(kA)	Primary relay	Fault current(kA)
1	4.0	2	3.0

Table 2: CT ratio for radial system

Relay no:	CT ratio	Relay no:	CT ratio
1	300/1	2	100/1

The results for radial system are shown in Table 3. Itis found that the maximum operating time of primary and backup relay is less than 1.0 second. The objective function so obtained is also bounded and is shown as the entry in the last row.

Tables 3: Time dial settings and operating times of radial system

Relays		TDS in sec		Operating time in sec	
P	B	Primary	Backup	Primary	Backup
1	2	[0.2592, 0.2593]	[0.1000, 0.1001]	[0.6818, 0.6819]	[0.2000, 0.2002]
Objective function				[0.4592, 0.4593]	

Similarly a 3-bus network shown in Figure 3 is analyzed to identify the primary/backup relay pairs. Then the values of short circuit current of backup and primary relays are calculated for the fault located near the primary relay of each pair. By using the values of the short circuit currents of backup and primary relays and pickup currents, we find using (6). The TDS values can range continuously from 0.1 to 1.1and CTI is assumed to be 0.2 seconds. The revised simplex method using intervals are used to calculate the values of TDS and the operating times of the primary and backup relays.

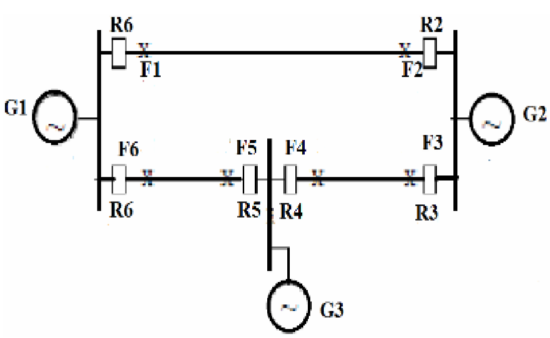

Figure. 3: A 3-bus test system

The results for 3-bus test system are shown in Table 4.It is found that the maximum operating time of primary and backup relay is less than 1.0 and 1.2 second respectively. The objective function so obtained is also bounded and is shown as the entry in the last row.

Table 4: Time Dial Settings and Operating Times of 3 Bus test Case

Relay Pairs		TDS	Operating times	
Backup	Primary		Backup	Primary
5	1	[0.0985, 0.0986]	[0.5021, 0.5022]	[0.2380, 0.2381]
4	2	[0.1802, 0.1803]	[0.4496, 0.4497]	[0.3117, 0.3118]
1	3	[0.1185, 0.1186]	[0.4707, 0.4708	[0.3021, 0.3022]
6	4	[0.1200, 0.1201]	[0.5117, 0.5118]	[0.3121, 0.3122]
3	5	[0.1511, 0.1512]	[0.4380, 0.4381]	[0.2707, 0.2708]
2	6	[0.1192, 0.1193]	[0.5121, 0.5122]	[0.2494, 0.2495]
Objective function			[1.6838, 1.6839]	

7. Conclusion

This paper presented an enhanced formulation of directional overcurrent relay coordination problem. The revised simplex linear programming with intervals is used. The proposed problem formulation requires a predetermines of the pickup current setting values for the optimal operating time determination. The optimal values of the operating time of the relays are bounded and the objective function is also bounded. The algorithm is tested for various systems, and is found to give satisfactory results in all the cases.

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